

**Wang and Plerou Reply:** In the preceding article [1], Tanaka and Machida (TM) comment on the bare values of the coupling constants in an effective field theory derived for the transport properties in the multilayer quantum Hall systems [2]. In Ref. [2], the field theory was derived by bosonization of a quantum quasi-1d fermionic theory using the nonabelian chiral anomaly. The latter arises following a finite chiral transformation of Dirac fermions, and is given by the corresponding Fujikawa Jacobian [3]. It was first pointed out by Roskies and Schaposnik [4], in the context of the Schwinger model, that a chiral rotation with a finite angle should be carried out by a sequence of infinitesimal rotations to produce the correct coupling constants in the anomalous terms. As correctly noted by TM, this was overlooked in Ref. [2]. It is, however, obvious that the precise values of the bare coupling constants are *totally irrelevant* for the analysis of the theory in Ref. [2], as the latter is based on the fixed point values of the field theory in two and three dimensions.

TM then argue in their comment [1] that the Fujikawa Jacobian becomes zero and thus no anomaly arises when a one parameter family of infinitesimal chiral transformations is used to carry out the finite chiral rotation in the nonabelian case considered in Ref. [2]. This claim is *incorrect*. Below, we will show explicitly that the correct chiral anomaly associated with the non-invariance of the fermion functional integral measure is obtained by the transformations used in Ref. [2]. We further prove that the chiral transformation suggested by TM is equivalent to the combined unitary and chiral rotations used in Ref. [2].

To make connection to the comment, we will consider the case of vanishing interlayer tunneling. The fermionic quantum theory becomes that of a  $U(2n)$  Hubbard chain at half-filling. For small hopping alternations  $\delta t \ll t$ , Eq. (5) in Ref. [2] can be written in terms of the Dirac spinors  $\Psi^T = (\psi_R, \psi_L)$ , and  $\bar{\Psi} = (\bar{\psi}_R, \bar{\psi}_L)\gamma_0$ ,

$$S = \text{Tr} \bar{\Psi} (\mathbf{I} \otimes \partial + m Q e^{-2iQ\Delta\theta\gamma^5}) \Psi. \quad (1)$$

Here,  $Q = u\Lambda u^\dagger$  with  $u \in U(2n)$  and  $\Lambda = \begin{pmatrix} \mathbf{I}_n & 0 \\ 0 & -\mathbf{I}_n \end{pmatrix}$ ,  $\partial = \gamma_\mu \partial_\mu$  with  $\gamma_0$  and  $\gamma_1$  given by the Pauli matrices  $\tau_x$  and  $\tau_y$ ,  $\gamma^5 = i\gamma_0\gamma_1$ ,  $m = -\Delta_0$ , and  $\Delta\theta = \delta t/\Delta_0$ .

As in Ref. [2], we make a unitary transformation  $\Psi \rightarrow u\Psi$ ,  $\bar{\Psi} \rightarrow \bar{\Psi}u^\dagger$  in Eq. (1), leading to,

$$S = \text{Tr} \bar{\Psi} (\mathbf{I} \otimes \partial + i\mathcal{A} + m\Lambda e^{-2i\Lambda\Delta\theta\gamma^5}) \Psi, \quad (2)$$

where the gauge field  $\mathcal{A} = -iu^\dagger \partial u$ . We now carry out the sequence of infinitesimal chiral transformations parametrized by the parameter  $t \in [0, 1]$ ,

$$\Psi \rightarrow U_5(t)\Psi, \quad \bar{\Psi} \rightarrow \bar{\Psi}U_5(t), \quad (3)$$

with  $U_5(t) = \exp(it\Lambda\phi\gamma^5)$  and  $\phi = (\Delta\theta + \pi/4)$ . The transformed action is  $S = \text{Tr} \bar{\Psi} \mathcal{D}_t \Psi + \ln \mathcal{J}_F$  where  $\mathcal{J}_F$  is the corresponding Jacobian and

$$\mathcal{D}_t = \mathbf{I} \otimes \partial + iU_5(t)\mathcal{A}U_5(t) - im\gamma^5 e^{2i\Lambda\phi(t-1)\gamma^5}. \quad (4)$$

The finite chiral rotation in Eq. (6) of Ref. [2] is built up by iterating Eq. (3). The chiral anomaly arises from the accumulated Jacobian of the transformations and is given by [5],

$$\ln \mathcal{J}_F = -\frac{1}{2\pi} \int_0^1 dt (i\phi\Lambda\gamma^5) \mathcal{D}_t^2, \quad (5)$$

where the operator  $\mathcal{D}_t$  is given in Eq. (4). Eq. (5) can be evaluated straightforwardly. Using the identity,  $\text{Tr} \epsilon_{\mu\nu} \Lambda \partial_\mu A_\nu = (i/4) \text{Tr} \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q$ , it leads to the topological term in the transformed action,

$$S_\theta = \frac{\sigma_{xy}^0}{8} \text{Tr} \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q, \quad (6)$$

where the bare coupling  $\sigma_{xy}^0 = \frac{1}{2\pi}(\pi + 4\Delta\theta + \sin 4\Delta\theta)$  is the *same* as the one obtained using nonabelian bosonization and in the comment [1].

TM used a seemingly different nonabelian chiral transformation,  $U'_5(t) = \exp(itQ\phi\gamma^5)$ , to directly bosonize Eq. (1). This leads to the transformed action  $S' = \text{Tr} \bar{\Psi} \mathcal{D}'_t \Psi + \ln \mathcal{J}'_F$  with

$$\mathcal{D}'_t = \mathbf{I} \otimes \partial + iU'_5(t)\partial U'_5(t) - im\gamma^5 e^{2iQ\phi(t-1)\gamma^5}, \quad (7)$$

and

$$\ln \mathcal{J}'_F = -\frac{1}{2\pi} \int_0^1 dt (i\phi Q \gamma^5) (\mathcal{D}'_t)^2. \quad (8)$$

We now prove that this is equivalent to the method used in Ref. [2] as described above. Notice that  $U'_5(t) = uU_5(t)u^\dagger$ . It is straightforward to show that  $\mathcal{D}'_t = u\mathcal{D}_t u^\dagger$ . As a result, the nonabelian chiral anomaly given by the Jacobians in Eq. (5) and Eq. (8) are identical, *i.e.*  $\ln \mathcal{J}_F = \ln \mathcal{J}'_F$ . Specifically,

$$\ln \mathcal{J}_F = \ln \mathcal{J}'_F = \frac{\sigma_{xx}^0}{8} \text{Tr} \partial_\mu Q \partial_\mu Q + \frac{\sigma_{xy}^0}{8} \text{Tr} \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q,$$

where  $\sigma_{xx}^0 = \frac{1}{2\pi} \cos^2 2\Delta\theta$ , and  $\sigma_{xy}^0 = \frac{1}{2\pi}(\pi + 4\Delta\theta + \sin 4\Delta\theta)$  is the same as the one given in Eq. (6).

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